

Observed Ω_b spectrum and meson-baryon molecular states

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We observe that four peaks seen in the high energy part of the Ω_b spectrum of the recent LHCb experiment are in remarkable agreement with predictions made for molecular Ω_b states stemming from the meson-baryon interaction, with an approach that applied to the Ω_c states gives rise to three states in good agreement with experiment in masses and widths. While the statistical significance of the peaks prevents us from claims of states at the present time, the agreement found should be an incentive to look at this experiment with increased statistics to give an answer to this suggestive idea.

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I. INTRODUCTION

The recent LHCb experiment on the Ω_b excited spectrum has reported four new peaks that have been associated with new excited Ω_b states, $\Omega_b(6316)$, $\Omega_b(6330)$, $\Omega_b(6340)$, and $\Omega_b(6350)$ [1]. The experiment has already stimulated theoretical work, and in Ref. [2], using QCD sum rules, the suggestion is made that the states correspond to ordinary $1P$ excitations of three quark states. Similar claims are made in Ref. [3] where decay modes of these states are discussed.

In this short paper, we discuss that these structures do not correspond to molecular states originated from the meson-baryon interaction. However, we call attention to four other peaks observed at higher energy, albeit with low statistics, which remarkably agree with predictions made with the molecular picture [4]. These peaks occur at 6402, 6427, 6468, and 6495 MeV, and the statistical significance is equivalent to that of the peak at 6315.64 MeV, which has given name to one of the states claimed, the $\Omega_b(6316)$. In Ref. [4], four states were found at 6405 MeV($\frac{1}{2}^-$), 6427 MeV($\frac{3}{2}^-$), 6465 MeV($\frac{1}{2}^-$), and 6508 MeV($\frac{1}{2}^-$, $\frac{3}{2}^-$), which well match the position of the peaks reported above. The widths of these states are smaller than 1 MeV, except for the 6465 MeV state that has a width of 2.4 MeV.

There is abundant work done on bottom baryons in the literature, mostly in quark models. A relativized quark model was earlier used in Ref. [5], and the recent progress detecting many such states has motivated much theoretical work recently. Nonrelativistic quark models are used in Refs. [6–9], full calculations with Faddeev equations are done in Ref. [10], a relativistic quark model is used in Refs. [11–13], heavy quark effective theory is used in Ref. [14], the quark pair creation model is used in Refs. [3,15,16], the relativistic flux tube model is used in Ref. [17], the color hyperfine interaction is used in Refs. [18,19], the chiral quark model is used in Refs. [20–22], and chiral perturbation theory is used in Ref. [23]. QCD sum rules have also been much used to calculate the Ω_b spectrum and decay widths [24–32], and as usual, they have uncertainties in the masses of the order of 100 MeV or more. Lattice QCD calculations have also brought their share to this topic [33]. The QCD motivated hypercentral quark mode is used in Refs. [34,35]. Some reviews on this issue are given in Refs. [19,36–41], and an update of more recent works can be seen in Refs. [3,42].

Most of the works on quark models calculate the mass spectra of the states but do not evaluate decay widths. There are, however, some works devoted to this task, mostly using the 3P_0 model [3,8,9,43] or the chiral quark model [20,22,44,45].

By contrast to the rich spectra of states in the quark model, the molecular states of Ω_b nature are much fewer, and rather than 500 MeV quark excitation in the quark configurations, one is talking about a few MeV binding of the main components. Provided one can tune the few free parameters to some experimental data, the predictions are relatively accurate. In the present case, such relevant

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information is provided by the analogous Ω_c states reported in Ref. [46].

In the next section, we discuss the origin of the masses for the molecular Ω_b states in connection with the spectrum of the related Ω_c states.

II. Ω_c AND Ω_b MOLECULAR STATES

It is very interesting to compare the spectrum of the Ω_c states reported by the LHCb [46] with that of the Ω_b states. In Ref. [46], five narrow states were observed and named $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$. Three of these states were reproduced in Ref. [47], both in the mass and the width, the $\Omega_c(3050)$, $\Omega_c(3090)$, and $\Omega_c(3119)$ states. The states were obtained by solving the Bethe-Salpeter equation in the coupled channels $\Xi_c \bar{K}$, $\Xi'_c \bar{K}$, ΞD , $\Omega_c \eta$, ΞD^* , $\Xi_c \bar{K}^*$, $\Xi'_c \bar{K}^*$, $\Xi_c^* \bar{K}$, $\Omega_c^* \eta$, and $\Xi^* D$, with an interaction based on the exchange of vector mesons extending the local hidden gauge approach [48–52] to the charm sector. The meson-baryon loop functions were regularized using the cutoff method, with $q_{\max} = 650$ MeV, the maximum value of the modulus of the three-momentum in the loop.

The scattering matrix for transitions between channels is given by

$$T = [1 - VG]^{-1}V, \quad (1)$$

with V the transition potential which is found as

$$V_{ij} = D_{ij} \frac{1}{4f^2} (p^0 + p'^0), \quad (2)$$

with f the pion decay constant $f = 93$ MeV, p^0 , p'^0 the energies of the initial and final mesons, respectively, and D_{ij} coefficients which are evaluated in Ref. [47]. The couplings of the states obtained and the wave function at the origin were evaluated, which allowed us to identify the most important coupled channel for each state.

In Table I, we show the results reported in Ref. [47], together with the main channel for each state, its threshold mass, and the diagonal D_{ii} coefficient for this channel. We can see that $\Omega_c(3050)$ is associated with a $J^P = \frac{1}{2}^-$ state at 3054 MeV, which is mostly a $\Xi'_c \bar{K}$ state bound by about

20 MeV. The other $\frac{1}{2}^-$ state at 3091 MeV is associated with $\Omega_c(3090)$. It is mostly a ΞD state bound by about 94 MeV. Finally, we have a $\frac{3}{2}^-$ state at 3125 MeV, which we associate with $\Omega_c(3119)$. It is mostly built up from the $\Xi_c^* \bar{K}$ channel and is bound by about 17 MeV. In addition, we also put in Table I a state found at 3221 MeV, degenerate in $\frac{1}{2}^-$, $\frac{3}{2}^-$, which couples mostly to the ΞD^* component and is bound by 106 MeV. We can see that the 3054 and 3125 MeV states are moderately bound by about 20 MeV, while the other two states are bound by about 100 MeV. The reason for this difference is the strength of the interaction represented by the D_{ij} coefficient of Eq. (2), which is double the other for the two most bound states.

The two $\frac{1}{2}^-$ states of Table I were earlier obtained in Ref. [53], with basically the same mass and width and the same main component, using an extension of the chiral unitary approach, with an interaction as the one in Eq. (2), which gave the same coefficients for the diagonal transitions. The $\frac{3}{2}^-$ state was not obtained because the Ξ_c^* baryon was not considered in the coupled channels. A different approach was used in Ref. [54] using an $SU(6)_{\text{lsf}} \times SU(2)$ heavy quark spin symmetry (HQSS) extension of the Weinberg-Tomozawa interaction, combining $SU(6)$ light quark spin-flavor symmetry and heavy quark spin symmetry for the heavy quarks. This approach leads to similar qualitative features, but depending on the regularization, it gives rise to more bound states. An approach based on meson exchange is used in Ref. [55] where a $\frac{3}{2}^-$ Ω_c bound state is found that couples mostly to $\Xi_c^* \bar{K}$, as in our case. Other works assuming molecular nature for the Ω_c states are Refs. [56–58], but there, only the ΞD (also ΞD^* in Ref. [58]) components are considered, and the decay modes are studied. The results in Table I are obtained with all coupled channels, and using an interaction which has proved very successful reproducing the LHCb pentaquark states [59] in Ref. [60].

One could apply arguments of HQSS [61–63] to relate the states of Table I with their analog ones of Ω_b , changing a quark c by the quark b . Instead of this, we study the molecular Ω_b states in Ref. [4] in the same footing as the Ω_c states, exchanging vector mesons to generate the interaction that is used together with the Bethe-Salpeter equation and the corresponding coupled channels. Yet, the approach

TABLE I. Experimental Ω_c states from Ref. [46] and theoretical predictions from Ref. [47]. The main channel for each state is shown together with its threshold mass and the diagonal D_{ii} coefficient for this channel. In brackets is the width of the state. Mass units are in MeV.

	$\Omega_c(3050)$	$\Omega_c(3090)$	$\Omega_c(3119)$	
Experimental state	$[0.8 \pm 0.2 \pm 0.1]$	$[8.7 \pm 1.0 \pm 0.8]$	$[1.1 \pm 0.8 \pm 0.4]$	
Ref. [47]	3054($\frac{1}{2}^-$) [0.88]	3091($\frac{1}{2}^-$) [10.2]	3125($\frac{3}{2}^-$) [0]	3221($\frac{1}{2}^-$, $\frac{3}{2}^-$) [0]
Main channel	$\Xi'_c \bar{K}$	ΞD	$\Xi_c^* \bar{K}$	ΞD^*
Threshold mass	3074	3185	3142	3327
D_{ii}	-1	-2	-1	-2

TABLE II. Peak energies from Ref. [1] at the higher energy region and theoretical predictions from Ref. [4]. The main channel for each state is shown together with its threshold mass and the diagonal D_{ii} coefficient for this channel. Mass units are in MeV.

Experimental peaks	6402	6427	6468	6495
Ref. [4]	6405($\frac{1}{2}^-$)	6427($\frac{3}{2}^-$)	6465($\frac{1}{2}^-$)	6508($\frac{1}{2}^-, \frac{3}{2}^-$)
Main channel	$\Xi'_b \bar{K}$	$\Xi_b^* \bar{K}$	$\Xi \bar{B}$	$\Xi \bar{B}^*$
Threshold mass	6431	6451	6598	6643
D_{ii}	-1	-1	-2	-2

respects HQSS, since in the transitions where light vectors are exchanged, the heavy quarks are spectators, and when heavy vectors are exchanged, the terms are suppressed in the heavy quark mass counting. This symmetry refers to the V_{ij} transition potentials, but it was discussed in Ref. [64] that extra caution should be exerted when regularizing the loops, which in the cutoff regularization method turned into using the same cutoff in the charmed or bottom sectors [65,66].

With all these considerations in mind, the Ω_b states were studied in Ref. [4] with the coupled channels $\Xi_b \bar{K}$, $\Xi'_b \bar{K}$, $\Omega_b \eta$, $\Xi \bar{B}$, $\Xi_b^* \eta$, $\Xi^* \bar{B}$, $\Xi \bar{B}^*$, $\Xi_b \bar{K}^*$, and $\Xi'_b \bar{K}^*$ using the same cutoff ($q_{\max} = 650$ MeV) as for the Ω_c states, and the results are summarized in Table II, together with the potential experimental states extracted from the peaks at high energy in the Ω_b spectrum of Ref. [1].

Unlike the case of the Ω_c , there is not much work on molecular Ω_b states. One exception is the work of Ref. [67] where, using $SU(6)_{lsf} \times SU(2)$ HQSS symmetry, a sextet is obtained with which the $\Sigma_b(6097)$ and $\Xi_b(6227)$ states are associated, and extrapolating the masses predicts an Ω_b state at 6360 MeV.

The features of the states obtained are nearly a copy of what is found for the Ω_c states. We find a $\frac{1}{2}^-$ state at 6405 MeV which couples mostly to $\Xi'_b \bar{K}$ and is bound by about 26 MeV. Another state of $\frac{3}{2}^-$ at 64427 MeV couples mostly to $\Xi_b^* \bar{K}$ and is bound by about 24 MeV. On the other hand, the $\frac{1}{2}^-$ state at 6465 MeV couples mostly to $\Xi \bar{B}$ and is bound by about 133 MeV, while the $\frac{1}{2}^-, \frac{3}{2}^-$ state at 6508 MeV couples mostly to $\Xi \bar{B}^*$ and is bound by about 135 MeV. The main channels of the states found are the same as in Table I changing a c quark by a b quark and the binding energies are very similar. We should note that the order of energies of the $\Xi_b^* \bar{K}(\frac{3}{2}^-)$, $\Xi \bar{B}(\frac{1}{2}^-)$ and $\Xi_c^* \bar{K}(\frac{3}{2}^-)$, $\Xi D(\frac{1}{2}^-)$ states is reversed, but this is not due to different bindings but to the masses of the components. The bindings are similar in both cases. Once again, we see that two of the states are moderately bound and they correspond to a diagonal interaction with $D_{ii} = -1$, while the other two states, which are more bound, correspond to $D_{ii} = -2$, with double strength in the interaction.

As to the width of the states that we get, we have $\Gamma \simeq 0$ MeV for the 6405 MeV($\frac{1}{2}^-$), 6427 MeV($\frac{3}{2}^-$), and 6508 MeV($\frac{1}{2}^-, \frac{3}{2}^-$) states, and $\Gamma \simeq 2.4$ MeV for the 6465 MeV($\frac{1}{2}^-$) state. The peaks that we refer to in Ref. [1] are very narrow, and by comparison to the $\Omega_b(6316)$, which has a width $\Gamma_{\Omega_b} < 2.8$ MeV [1], we can reasonably guess that should these peaks be confirmed as states in future runs, the width is also smaller than 2.8 MeV, which would be in agreement with our findings.

The measurement of the widths is also relevant and could be a decisive factor to distinguish molecular states from ordinary $3q$ states. In this sense, it is worth recalling that for the Ω_c molecular states associated with the experimental ones, the agreement of the widths with experiment was remarkable, as one can see in Table I. In this sense, it is worth recalling the result of $\Gamma_{\Omega_b(3/2^-)} = 58_{-33}^{+65}$ MeV of Ref. [30] for a state at 6460 MeV and widths evaluated for other states that have a very wide margin of variation.

There is one more reason that we can provide to argue that the states reported in Ref. [1] are not of molecular nature. We can artificially increase q_{\max} to match one of the states and see what we obtain for the other ones. We find that with $q_{\max} = 830$ MeV, we get a $\frac{1}{2}^-$ state at 6317 MeV that matches the $\Omega_b(6316)$ in mass, but the width becomes 13 MeV, in disagreement with the width smaller than 2.8 MeV claimed in Ref. [1]. Also, the binding of the $\Xi'_b \bar{K}$ component would be 115 MeV, which we find too big for the strength of the interaction with coefficient $D_{ii} = -1$ compared with the 20 MeV binding of the equivalent Ω_c state. Furthermore, we obtain then for the masses of the other states 6358 MeV($\frac{1}{2}^-$), 6362 MeV($\frac{1}{2}^-, \frac{3}{2}^-$), and 6375 MeV($\frac{3}{2}^-$), none of which matches the mass of the reported Ω_b states.

It is clear that the peaks seen in the Ω_b spectrum reported in Table II do not have enough statistics to make a claim of states, but we should emphasize that the statistical significance is the same as for the $\Omega_b(6316)$ claimed in Ref. [1]. Yet, it is difficult to think that the impressive agreement of the energy of these four peaks with the predictions of Ref. [4] is a pure coincidence. This observation should be reason enough to look into the experiment and come back with more statistics to corroborate this ansatz or refute it, which is the main purpose of the present work.

III. COMPARISON WITH THE RESULTS OF QUARK MODELS

It is interesting to establish a comparison with the results of quark models. Most of the recent work on quark models tries to describe the reported Ω_b states of Ref. [1]. The majority of the works obtain Ω_b states around 6350 MeV for the $1P$ excitations with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$. The widths depend much on the models. In Ref. [20], a width of 143 MeV is obtained for one of the low-lying $\frac{1}{2}^-$ states, while 95 MeV is

obtained in Ref. [44]. For another $\frac{1}{2}^-$ state, Ref. [44] obtains 49 MeV, but both these works give widths smaller than 2 MeV for the $\frac{3}{2}^-$ states. Similar values are obtained in Ref. [22] and smaller values in Refs. [3,43]. In Ref. [3], one of the $\frac{1}{2}^-$ states has, however, a width as large as 1000 MeV, which, as discussed there, would make the state unobservable, and even bigger values are found within the QCD sum rule calculation of Ref. [30].

The states that we propose appear from 6400 to 6500 MeV and are not those reported in Ref. [1]. The first lesson one learns is that the first molecular states that appear in Ref. [4] with $\frac{1}{2}^-, \frac{3}{2}^-$ are very different from those of the quark model, which appear from $1P$ excitation roughly about 100 MeV below those of Ref. [4]. Then in the quark models, one has to go to the $2S$ excited states, which are in the region of 6450 MeV [9,11,13,14,43] to eventually match the energies of the peaks that we propose as states. Some models predict the masses a little lower, below 6400 MeV [10,68], and the QCD sum rules of Ref. [69] above 6000 MeV, although compatible with values around 6450 MeV considering the uncertainties. Then, these states would have positive parity, unlike the molecular states that have negative parity. Here, one can see a very distinctive dynamical character for these models, and measuring the quantum numbers of these states can clearly discriminate between them.

As to the widths of the states, according to Ref. [43] the $2S$ quark model states are expected to have widths of around 15 MeV, but there are also differences between the different models.

In our molecular model, one can obtain the widths from the couplings to the different channels. This is a peculiar feature of the coupled channels approach, since different orthogonal combinations of the channels are obtained as eigenstates, and the couplings vary much from one state to another. The case is different for the results that one would obtain from the 3P_0 decay model in quark configurations.

In this sense, we can go to Ref. [4] and, by using the couplings of the states obtained there to the open channels, we can evaluate the partial decay widths. We have the decay width given by

$$\Gamma = \frac{1}{2\pi} \frac{M_{\text{fin}}}{M_{\text{in}}} |g|^2 p_m, \quad (3)$$

where M_{fin} , M_{in} are the mass of the final baryon in the decay channel and the mass of the obtained Ω_b state, and p_m is the meson momentum of the meson-baryon decay channel. We find that the 6405 MeV state is only above the $\Xi_b \bar{K}$ threshold and only decays into that channel. The 6465 MeV state decays into $\Xi_b \bar{K}$ and $\Xi'_b \bar{K}$. The state at 6427 MeV couples to $\Xi_b^* \bar{K}$ and other channels that are all above this energy. Similarly, the state at 6508 MeV couples to $\Xi \bar{B}^*$ and other channels which are all closed. Using Eq. (3) and the couplings of Ref. [4], we obtain the partial

TABLE III. Partial decay widths of the predicted Ω_b states to decay channels (units in MeV).

State	$\Xi_b \bar{K}$	$\Xi'_b \bar{K}$	Total
6405.2	0.024	...	0.024
6427
6465	1.67	0.755	2.42
6508

decay widths of Table III. We can see that within our model, that accounts for the main components in the coupled channels, the width of the 6427 and 6508 states is zero. Comparing that with the largest width that we obtain of 2.42 MeV, we can reasonably expect that the widths of these states should be smaller than 1 MeV. The 6405 state only decays into $\Xi_b \bar{K}$, and since the coupling of the state to this channel is very small, the width that we obtain is a mere 0.024 MeV. On the other hand, we can split the total width of the 6465 state into two partial decays, one with 1.67 MeV to the $\Xi_b \bar{K}$ channel and another one with 0.75 MeV into the $\Xi'_b \bar{K}$ channel. This is valuable information that could be contrasted with experiment in future runs. The widths, the partial decay widths, and the spin and parity of these possible states would be very different from any quark model prediction. By comparison, the widths evaluated with the 3P_0 models from the quark model states predict widths of the order of 15 MeV for $2S$ states with masses 6483 and 6495 MeV in Ref. [43]. In Ref. [3], the $2S$ state of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ have widths around 50 MeV. On the other hand, in Ref. [22] the widths obtained for two $2S$ states are smaller than 2 MeV. With these different results in different quark models, it is difficult to assert the values of the width for the $2S$ states, but they all share that the parity is positive in contrast with the negative parity of the states that we predict in that region. Our predictions for the widths in Table III are quite stable, and it would be most interesting to see if future runs of the experiments can measure these rates.

IV. CONCLUDING REMARKS

We showed that the results obtained in Ref. [4] for molecular Ω_b states from the meson-baryon interaction in coupled channels constructed by analogy to the Ω_c states of Ref. [47], which agree with the experimentally observed states [46], give rise to four states with energies in remarkable agreement with some peaks that can be observed in the high energy part of the Ω_b experimental spectrum [1]. While the statistical significance is poor, we noticed that it is about the same as for the first of the Ω_b states claimed in Ref. [1]. While we refrained from claiming that these peaks correspond to Ω_b states, the agreement found is reason enough to suggest increasing the statistics and looking in detail in the region where these peaks have appeared to hopefully find an answer to the suggestion made in this work.

We also made a comparison of our results with different quark models, concluding that the recently observed Ω_b states could well be ordinary $1P$ excitations of three quark systems, while the states that we predicted cannot accommodate these states. On the other hand, the quark models predicted states around 6450 MeV, where we found our $\frac{1}{2}^-$, $\frac{3}{2}^-$ molecular states, but as $2S$ excitations, which have positive parity. We also evaluated partial decay widths to different channels, and while there were discrepancies in the widths of these states predicted by different quark models, it was clear that the predictions were quite different from the results that we obtained. It is thus clear that precise measurements in that region, with enough statistics to claim new states and determine the widths, partial decay widths, and eventually the spin and parity of the states, could

clearly discriminate between the excited conventional three quark states and the molecular states that we proposed.

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